

# Evaporation and Combustion of a Single-Component Fuel in a Chamber of a Liquid Rocket Engine, LRE

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## Nomenclature

$a$	= coefficient in the formula $c_d$
$A$	= rate of evaporation parameter, $m^{3/2}/\text{sec}$
$b$	= coefficient in the Nusselt criterion formula
$B$	= $c_p \Delta T / l$
$c_p$	= heat capacity of vapors, $\text{kcal/kg-deg}$
$c_d$	= coefficient of the aerodynamic resistance of the drops
$d_K$	= flowing diameter of the drops, $m$
$d_{z_0}$	= average initial Sauter mean diameter, $m$ .
$D$	= $12 \frac{b \lambda \ln(1+B)}{a c_p \eta \gamma}$
$\eta$	= dynamic viscosity of gases, $\text{kg-sec}/m^2$
$f$	= aerodynamic force, $kg$
$g$	= $9.81 m/\text{sec}^2$ —acceleration force due to gravity
$G$	= rate of fuel injection, $kg/\text{sec}$ .
$G_p$	= weight amount of vapors, formed per sec, $kg/\text{sec}$
$\gamma_T$	= density of the liquid fuel, $kg/m$
$\gamma_0$	= density of delayed-reaction combustion products, $kg/m$
$j$	= acceleration of gases, $m/\text{sec}$
$j_K$	= acceleration of drops
$l$	= heat of evaporation of the fuel, $\text{kcal/kg}$
$m_K$	= mass of flowing drops, $kg$
$Nu$	= Nusselt number
$p_0$	= pressure of gases in the chamber, $kg/m^2$
$q$	= mass flux of substances evaporating from the drop's surface, $kg/\text{sec-m}^2$
$r$	= the flowing drop's radius, $m$
$Re$	= Reynolds number
$S$	= cross section of the chamber of LRE, $m^2$
$S_K$	= the drop's cross-sectional area, $m^2$
$t$	= time, $\text{sec}$
$T_0$	= temperature of delayed-reaction gases, $^{\circ}\text{K}$
$T_{\text{boil}}$	= fuel's boiling temperature, $^{\circ}\text{K}$
$\Delta T'$	= $T_0 - T_{\text{boil}}$
$u$	= velocity of drops relative to that of gases, $m/\text{sec}$
$v$	= drop velocity relative to the chamber's walls
$w$	= velocity of the gases relative to the chamber
$x$	= path traversed by the drops, $m$
$x$	= path traversed by gases
$z$	= relative amount evaporated

**THE THEORY** regarding operating conditions in a combustion chamber of an LRE has until now been exposed insufficiently in the literature. In 1958, an article by Sodha (1)<sup>1</sup> was published, in which final solutions are presented in the form of Bessel and Neuman functions, but the initial conditions contain relationships that are far from the truth. It is assumed, for instance, that the rate of radius decrease of an evaporating drop is constant, whereas the index of spraying uniformity is equal to unity. It seems to us that the evaporation theory of drops in a gas stream, developed by Frank-Kamenetskyi (2) and supplemented by certain plausible assumptions, enables one to calculate the main parameters and chamber dimensions of an LRE with an accuracy sufficient for engineering purposes, presenting the final results in the form of customary algebraic equations.

In the present paper, the following working model of the process occurring in the chamber of an LRE is adopted.

A single-component fuel or a previously prepared liquid fuel mixture and oxidizing agent are injected into the chamber in the form of discrete drops that are ignited when contact with hot combustion products is effected (Fig. 1a). Combustion takes place only in the gaseous phase, since the liquid's boiling point is usually considerably lower than the ignition temperature. Vapor jackets formed around the drops absorb the heat of evaporation and superheating of the vapors, retarding the vapor-forming process. The combustion products and formed vapors accelerate, overtaking the drops and blowing vapor tufts away from them (Fig. 1b). Vapor combustion takes place according to laws of microturbulent burning (3). At high temperatures, which prevail in LRE chambers, the processes of microturbulent diffusion and chemical oxidation usually occur at faster rates than the liquid's evaporation. Therefore, evaporation of a liquid fuel appears to be, from our point of view, the limiting process, through which overall burning rates, chamber parameters, and chamber dimensions are determined. This fact also comprises our first assumption.

The microturbulent combustion process proceeds very rapidly. Therefore, we shall assume that the temperature of combustion products is constant along the entire length of the chamber. This constitutes our second assumption.

Prior to the beginning of noticeable liquid evaporation, the chamber is filled with combustion products, the velocity of which in the fuel supply zone is close to that of liquid outflow. According to the rise in the degree of evaporation, new gases are intermixed, and the cross-sectional average velocity of the gases increases, while the static pressure along the chamber's axis decreases slightly. In drops lagging behind the combustion products, there arises a relative velocity and aerodynamic force, striving to entrain the drops behind the gases. Subsequently, the drops start to move rapidly, and their acceleration gradually approaches that of the gases. The presence of a relative velocity intensifies the heat and mass transfer processes, increasing the rate of evaporation and combustion of the drops. Therefore, the exchange processes between drops and combustion products are characterized by Nusselt numbers which considerably exceed the value of 2. The smaller the initial Sauter mean diameter, the less the acceleration of the drops differs from that of the flow of the gases; toward the end of the evaporation, the accelerations become equal, and the relative velocity of the drops falls to zero.

The parameters and dimensions of an LRE chamber can be found by the numerical integration method of the motion and evaporation equations of the drops, taking into consideration, as Frank-Kamenetskyi (2) has shown, that, in the combustion zone, the liquid's surface temperature is close to the boiling point (Fig. 2).

It is possible to set up a simple theory regarding the operating conditions in an LRE chamber, assuming that the evaporation of liquid fuel from the beginning to end proceeds sufficiently slowly so that the droplet acceleration is close to that of the gases. This constitutes our last assumption. In the initial stages of the process, this assumption leads to overestimated rates of mass transfer. However, within a short distance down the length of the chamber, this effect practically disappears.

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<sup>1</sup>Numbers in parentheses indicate References at end of paper.

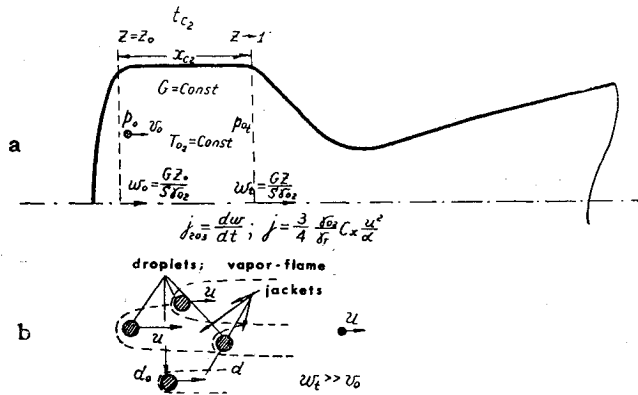


Fig. 1 a) Design model of operating conditions in an LRE  
b) vapor firing of drop tufts

Let us mention that the equality regarding the acceleration of drops and gases is also postulated by authors who assume that the burning drops in the cylindrical chamber are stationary relative to the rapidly moving surrounding gases. In reality, however, such a state is impossible, inasmuch as the cause that leads to the drops' acceleration is absent.

### Evaporation and Combustion of Drops, the Acceleration of Which Is Close to That of the Flow

$G$  kg/sec of fuel are introduced into a chamber of an LRE having a cross section  $S$ . The pressure of the gases in the combustion chamber  $p_0$  is determined by the gas flow, the chamber temperature  $T_0$ , and the cross section of the nozzle throat. The temperature of the combustion products  $T_0$  is determined by the nature of the reactants and by the fuel-oxidizer ratio.

Let us assume that the temperature and pressure are constant in the entire chamber:

$$T_0 = \text{const} \quad p_0 = \text{const}$$

Density of the combustion products  $\gamma_0$  is found according to the equation of state:

$$\gamma_0 = \text{const}$$

The cross-sectional average velocity of combustion products  $w$  is proportional to the amount evaporated, since it is assumed that the formed vapors burn rapidly:

$$w = \frac{G}{S\gamma_0} z \quad [1]$$

Relative evaporation capacity is defined as the ratio of the formed vapors to the fuel mass, introduced simultaneously into the combustion chamber:

$$z = \frac{G_p}{G} \quad [2]$$

The initial velocity  $w_0$  of the combustion products can be calculated through insertion of a certain initial evaporation capacity  $z$ :

$$w_0 \approx v_0 = \frac{G}{S\gamma_0} z_0$$

Acceleration of combustion products in the cylindrical chamber is determined by the evaporation rate  $dz/dt$  and the flow of fuel:

$$j = \frac{dw}{dt} = \frac{G}{S\gamma_0} \frac{dz}{dt} \quad [3]$$

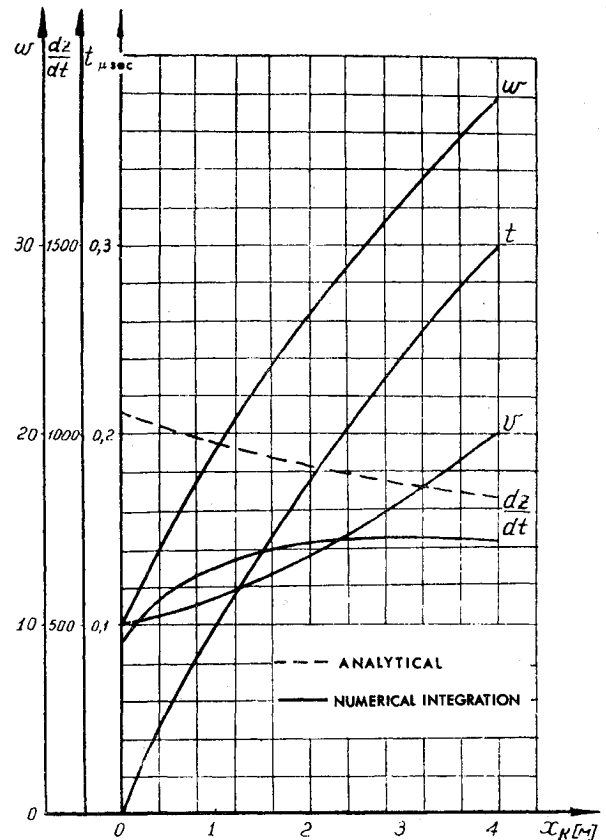


Fig. 2 Parameters of an LRE chamber as a function of the combustion zone length arrived at by numerical integration:  $z = 0.05$ ;  $C = 1$ ;  $l = 100$ ;  $\Delta T = 2500^\circ$ ;  $\lambda = 3 \times 10^{-5}$  kcal/m-sec-deg;  $\eta = 7 \times 10^{-6}$  kg sec/m<sup>2</sup>;  $d_0 = 50$ ;  $a = 29$ ;  $b = 0.54$ ;  $B = 25$ ;  $A = 0.0004$  m<sup>3/2</sup>/sec

Drops formed in the spraying of the liquid fuel are carried away by the stream of combustion products. The acceleration of gases is determined by the negative pressure gradient. Acceleration of drops is caused by an aerodynamic force that emerges because the velocity of the drops is lower than the velocity of the surrounding combustion products.

The acceleration of each drop  $j_K$  is directly proportional to the aerodynamic force and inversely proportional to the mass:

$$j_K = \frac{f}{m_K} = c_x \frac{S_K}{m_K} \frac{\gamma_0 u^2}{2} \quad [4]$$

where  $u = w - v$  = relative speed of the drop.

The ratio of the cross-sectional area of the drop to its mass is equal to

$$\frac{S_K}{m_K} = \frac{3/2}{\gamma_T d_K} \quad [5]$$

In studying motion and evaporation of a series of drops that have dissimilar dimensions, formed during spraying, it is reasonable to make use of an average Sauter diameter  $d_{s0}$ , instead of the true diameter of each drop.

Let us assume that, in the entire range of Reynolds numbers encountered in the motion of drops, the coefficient of resistance is expressed by the Vyrubov formula (4):

$$c_x = \frac{a}{\sqrt{Re}} = a \sqrt{\frac{g\eta}{u\gamma_{og} d_K}} \quad [6]$$

The numerical coefficient  $a$  depends on the degree of drop deformation. For a sphere with  $Re \approx 100$ ,  $a$  equals 11 (5). For drops flattened under the action of aerodynamic forces,  $a$  equals 14 to 30.

The relative velocity  $u$ , with which the drops move under the action of an aerodynamic force with an acceleration  $j$ , as follows from Eqs. [4-6] is equal to

$$u = \left(\frac{4}{3a}\right)^{2/3} d_K^3 \sqrt[3]{\frac{\gamma_T^2 J_K^2}{g\eta\gamma_{og}}} \quad [7]$$

Within a certain time after the beginning of combustion, the acceleration of drops approximates that of combustion products  $j$ . In this case, taking Eq. [3] into consideration, we obtain

$$u = \left(\frac{4}{3a}\right)^{2/3} d_K^3 \sqrt[3]{\frac{\gamma_T^2 G^2}{g\eta\gamma_{og}^3 S^2} \left(\frac{dz}{dt}\right)^2} \quad [8]$$

The Reynolds and Nusselt numbers for drops, the acceleration of which approaches that of the flow of the gases, can be found by employing Vyrubov's formula (4):

$$Re = \frac{u\gamma d_K}{g\eta} = \left(\frac{4}{3a}\right)^{2/3} d_K^2 \sqrt[3]{\left(\frac{\gamma_T G}{g^2\eta^2 S} \frac{dz}{dt}\right)^2} \quad [9]$$

$$Nu = b\sqrt{Re} = \left(\frac{4}{3a}\right)^{1/3} b d_K \sqrt[3]{\frac{\gamma_T G}{g^2\eta^2 S} \frac{dz}{dt}} \quad [10]$$

The empirical coefficient  $b$ , based on Vyrubov's experiments, is equal to 0.54.

The drops move in a stream of hot combustion products, the average temperature of which is much higher than the boiling temperature of the liquid. In this case, as Frank-Kamenetskiy (2) has shown, the equilibrium temperature of the surface of the drop is close to the boiling temperature of the liquid, and the evaporation rate is expressed by  $\epsilon$  logarithmic formula

$$q = \frac{Nu\lambda}{c_p d_K} \ln\left(1 + c_p \frac{T_{og} - T_{boil}}{l}\right) \quad [11]$$

Inserting the designation

$$B = c_p \frac{T_{og} - T_{boil}}{l} \quad [12]$$

The velocity responsible for the radius diminution of the drops

$$\frac{dr}{dt} = -\frac{q}{\gamma_T} = \frac{Nu\lambda}{c_p d_K \gamma_T} \ln(1 + B) \quad [13]$$

is directly proportional to the Nusselt number and inversely proportional to its diameter.

Substituting [10 and 12] in [11], we find that the flow of evaporated substances from the surface of the drops, the acceleration of which approaches that of the gases' flow, does not depend on the drop diameter

$$q = b \sqrt[3]{\frac{4}{3a} \frac{\lambda}{c_p}} \ln(1 + B) \sqrt[3]{\frac{\gamma_T G}{g^2 \eta^2 S} \frac{dz}{dt}} \quad [14]$$

Temperature elevation and acceleration of gases causes the flow of evaporated substances to rise.

The rate of evaporation  $dz/dt$  is equal to

$$\frac{dz}{dt} = \frac{dz}{dr} \frac{dr}{dt} \quad [15]$$

Substituting Eqs. [2, 13, and 14] in [15], after transforming we obtain

$$\frac{dz}{dt} = \frac{A}{d_{z_0}^{3/2}} (1 - z) \quad [16]$$

Here the following designation is inserted:

$$A = \sqrt{\frac{4}{3a}} (6b)^3 \frac{\sqrt{G} \left[ \frac{\lambda}{c_p} \ln(1 + B) \right]^3}{g\eta\gamma_T} \text{ m}^{3/2}/\text{sec} \quad [17]$$

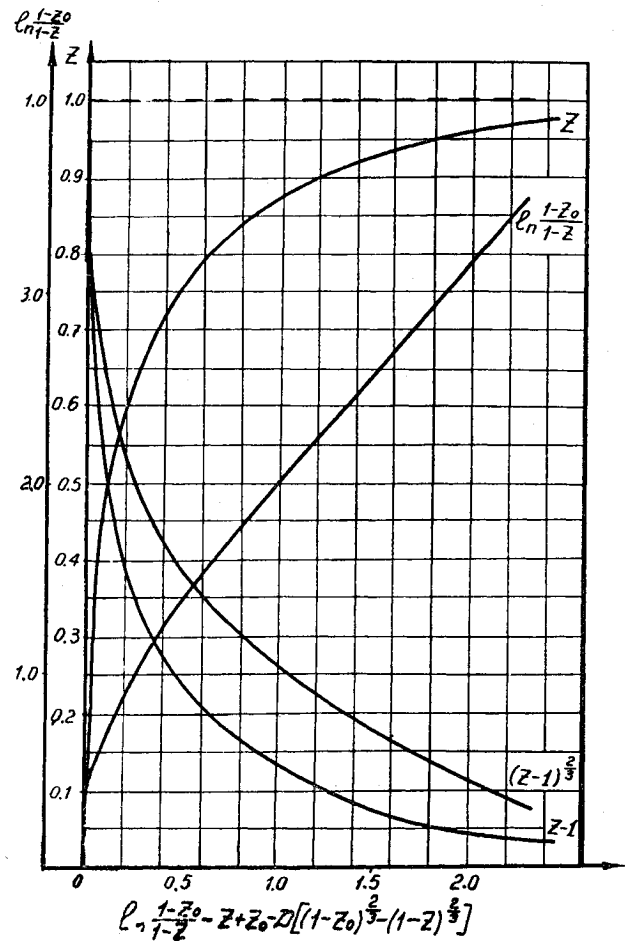


Fig. 3 Dimensionless parameters of an LRE combustion chamber operating on a single-component liquid fuel:  $z = 0.05$ ;  $D = 0.3$

The rate parameter  $A$ , which relates to evaporation and combustion, depends on the physical properties of the fuel and the combustion products.  $A$  has an order of magnitude of several ten thousandths  $\text{m}^{3/2}/\text{sec}$ . So, with  $G/S = 500 \text{ kg/sec-m}^2$ ,  $\Delta T = 2500^\circ$ ,  $c = 1 \text{ kcal/kg-deg}$ ,  $\lambda = 3 \times 10^{-5} \text{ kcal/sec-m-deg}$ ,  $\eta = 7 \times 10^{-6} \text{ kg-sec/m}^2$ ,  $a = 29$ ,  $b_s = 0.54$ ,  $\gamma_T = 1000 \text{ kg/m}^3$ :

$$A = \frac{\sqrt{\frac{4}{3 \cdot 29}} (6 \cdot 0.54)^3}{9 \cdot 8 \cdot 7 \cdot 10^{-6} \cdot 1000} \sqrt{500 \left[ \frac{3 \cdot 10^{-5}}{1} \ln\left(1 + 1 \frac{2500}{100}\right) \right]^3} = 0.0004 \text{ m}^{3/2}/\text{sec}$$

Substituting the evaporation rate value found in [3], we derive the acceleration of the hot gases:

$$j = \frac{AG}{d_{z_0}^{3/2} S \gamma_0} (1 - z) \quad [18]$$

Separating the variables in Eq. [16] and integrating within the limits from  $t = 0$  to  $t$  and from  $z_0$  to  $z$ , we can find the time as a function of combustion completeness:

$$t = \frac{d_{z_0}^{3/2}}{A} \ln \frac{1 - z_0}{1 - z} \quad [19]$$

The evaporation and combustion times of a series of liquid fuel drops is directly proportional to the average Sauter  $d_{z_0}$  spraying diameter to the power of  $\frac{3}{2}$  and inversely proportional to the evaporation parameter  $A$  (Fig. 3).

Substituting the found evaporation and combustion rate values in [8], we find that the relative velocity of the drops

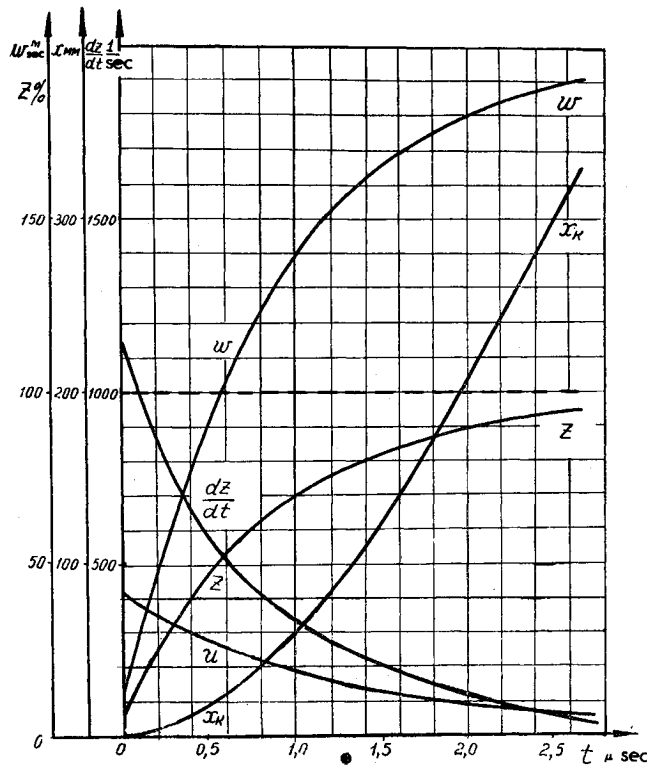


Fig. 4 Chamber parameters of an LRE as a function of combustion time of a single-component liquid fuel:  $G/S = 500$  kg/sec-m;  $d_{z0} = 50$ ;  $z_0 = 0.05$ ;  $c = 1$  kcal/kg-deg;  $\lambda = 3 \times 10^{-5}$  kcal/sec-m-deg;  $\eta = 7 \times 10^{-6}$  kg-sec/m<sup>2</sup>;  $a = 29$ ;  $b = 0.54$ ;  $l = 100$  kcal/kg;  $\Delta T = 2500^\circ$ ;  $\gamma_T = 1000$  kg/m<sup>2</sup>;  $P = 22$  kg/cm<sup>2</sup>

in an LRE chamber does not depend on the initial Sauter  $d_{z0}$  diameter, but it decreases rapidly with the increase of the relative evaporation capacity  $z$ :

$$u = \frac{8b}{a} \frac{\lambda}{c} \frac{G \ln(1+B)}{g\eta S \gamma_0} (1-z)^{2/3} \text{ m/sec} \quad [20]$$

With  $\lambda = 3 \times 10^{-5}$  kcal/sec-m-deg,  $c = 1$  kcal/kg-deg,  $G/S = 500$  kg/sec-m<sup>2</sup>,  $B = 25$ ,  $\eta = 7 \times 10^{-6}$  kg/sec-m<sup>2</sup>,  $a = 29$ ,  $b = 0.54$ ,  $\gamma_0 = 2.5$  kg/m<sup>3</sup>:

$$u = 42(1-z)^{2/3}$$

With temperature and pressure decrease of the combustion products  $T$  and  $p$ , respectively, the limiting relative velocity of the drops diminishes.

The velocity of combustion products is expressed by Eq. [1]. The velocity of the drops is equal to the geometrical difference

$$\bar{v} = \bar{w} - \bar{u}$$

$$v = \frac{G}{S \gamma_0} \left[ z - \frac{8b}{a} \frac{\lambda}{c} \frac{\ln(1+B)}{g\eta} (1-z)^{2/3} \right] \quad [21]$$

The path traversed by the drops during the evaporation and combustion times, that is, the calculated length of the chamber's cylindrical section, is expressed by the definite integral

$$x = \int_{z_0}^z v dt$$

Using Eqs. [16] and [21] we get

$$x = \frac{G d_{z0}^{3/2}}{A S \gamma_0} \int_{z_0}^z \left\{ \frac{z}{1-z} - \frac{8b}{a} \frac{\lambda}{c} \frac{\ln(1+B)}{g\eta \sqrt[3]{1-z}} \right\} dz$$

Hence

$$x = \frac{G d_{z0}^{3/2}}{A S \gamma_0} \left\{ \ln \frac{1-z_0}{1-z} - z_0 + z - 12 \frac{b}{a} \frac{\lambda}{c} \frac{\ln(1+B)}{g\eta} \times \right. \\ \left. [(1-z_0)^{2/3} - (1-z)^{2/3}] \right\} \quad [22]$$

The combustion zone length of the sprayed single-component liquid fuel is directly proportional to the maximum velocity of combustion products  $G/S \gamma$ , to the average Sauter spraying diameter to the power of  $3/2$ , whereas it is inversely proportional to the rate evaporation parameter  $A$  and depends on the value of the relative amount evaporated  $z$ . The first polynomial in figured brackets expresses the path traversed by combustion products; the second polynomial characterizes the lagging drops, stipulated by the existence of the relative velocity  $u$ .

With  $a = 29$ ,  $b = 0.54$ ,  $B = 25$ ,  $c = 1$  kcal/kg-deg,  $\lambda = 3 \times 10^{-5}$  kcal/sec-m-deg,  $\eta = 7 \times 10^{-6}$  kg-sec/m<sup>2</sup>:

$$D = 12 \frac{b}{a} \frac{\lambda}{c} \frac{\ln(1+B)}{g\eta} \approx 0.3$$

If  $G/S = 500$  kg/sec-m<sup>2</sup>,  $p_0 = 22$ ,  $d_{z0} = 50$ ,  $T_4 = 2500^\circ\text{K}$ ,  $R = 30$  m/deg, and the completeness of combustion given as  $z = 0.98$ , then, according to Fig. 3, the expression in the figured brackets of Eq. [22] is equal to 2.5,  $\gamma_0 = 3$  kg/m<sup>3</sup>, and  $A = 0.0004$  m<sup>3/2</sup>/sec. Then the calculated evaporation and combustion length is equal to

$$x = 2.5 \frac{G d_{z0}^{3/2}}{A S \gamma_0} = \frac{2.5 \cdot 500 (50 \cdot 10^{-6})^{3/2}}{0.0007 \cdot 3} = 0.3 \text{ m}$$

which corresponds to the combustion chamber length of real LRE (7).

It is clear that combustion chambers for two-component fuels should be of greater length than those of single component ones, since the evaporation and combustion processes in the latter case are accompanied by turbulent mixing processes.

With an elevation in the fuel injection rate, the dimensions of the sprayers usually increase. In addition to that, the sprayer deteriorates, and the necessary chamber length increases. Thus the chamber length of the V-2 engine was of the order of 0.4 m, whereas that of the LRE Rocketdyne F-1, having a rated traction of 680 tons, was of the order of about 0.8 m (8).

It is noteworthy that the stated method gives results which are close to reality only with sufficiently high evaporating capacities:  $z > 0.2$ , when a relative velocity  $u$ , close to the calculated one, is successfully attained. The primary parameters of the combustion chamber can be found by numerically solving the drops' motion and evaporation equations.

## Conclusions

An evaporation and combustion theory of a single-component liquid fuel in an LRE chamber is developed which indicates the following.

1) With a decrease of the drops' average Sauter diameter, the combustion-zone length of a single-component or of a premixed fuel in LRE chamber decreases. If the spray is very fine, the evaporation rate will become greater than the mass velocity due to microdiffusional turbulent combustion, and the forementioned theory will cease to approach reality.

2) By raising the temperature increment of gases during combustion, that is, by increasing the fuel's heat of reaction, the evaporation and combustion rates rise according to the logarithmic law, whereas the combustion time and necessary chamber length decrease.

3) With an increase of the liquid-fuel mass flow and heat conductivity of gases, the evaporation and combustion rates rise, but with an increase of gases' heat capacity, viscosity, and the fuel density, the evaporation and combustion rates decrease.

4) The calculated zone length for the evaporation and combustion of drops of a single-component or a previously intermixed liquid fuel is close to the length of an LRE chamber, as determined by an empirical method.

—Submitted February 2, 1960

### Reviewer's Comment

To the best of the reviewer's knowledge, this is the first published Russian analysis of a detailed model of the combustion process in a liquid propellant rocket motor. A steady-state, one-dimensional model involving two-phase flow (liquid droplets and gaseous reaction products) is considered. The distribution in droplet sizes is neglected, and vaporization is assumed to be the rate-controlling process.

The author's study apparently was inspired by the work of Sodha (Ref. 1 of the paper); Il'yashenko appears to be unaware of the more significant publications in English (1-6) which began with the classical analysis of Probert (1) in 1946 and included a wealth of NASA research (5). The author's model resembles that of Spalding (3), but his treatment retains the Lagrangian aspects of Sodha's analysis instead of the simpler and more transparent Eulerian approach of Spalding.

The author's approximations appear to be less realistic than those of Spalding. The treatment of gas and droplet accelerations is particularly questionable and does not seem to pay proper respect to Newton's second law. It is curious how, while explicitly assuming that the acceleration of the droplets is equal and not opposite to that of the gas, the author finds that the relative velocity of the gas and the droplets decreases as they travel downstream. Nevertheless, the author obtains the usual results, which are virtually unavoidable in one-dimensional analyses (7): the required length of the combustion chamber increases as the injection

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velocity and droplet diameter increase and decreases as the evaporation rate increases.

It is worth mentioning that the correlation between drag coefficient and Reynolds number used by the author (Eq. [6]) was obtained from early Russian work (Ref. 4 of the paper) and appears to be less accurate than the correlations currently in use (6) in this country. Another interesting observation is that the only data quoted by the author in his experimental comparison is for the German V-2 and the Rocketdyne F-1.

Since the author's analysis is second-rate by Western standards, we may conclude that either 1) other, more realistic analyses exist in Russia or 2) successful rocket motor design does not require analyses of this kind.

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## Equations of the Precessional Theory of Gyroscopes

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**E**ULER'S equation for gyroscopic motion, obtained by using the theorem of moments, is as follows

$$\begin{aligned} A(\dot{p} - qr') + Hq &= M_x \\ A(\dot{q} + pr') - Hp &= M_y \\ \dot{H} &= M_z \end{aligned} \quad [1]$$

where  $z$  is the gyroscope axis,  $x$  and  $y$  are the axes in the equatorial plane of the inertia ellipsoid, constructed for the suspension point;  $A$  is the gyroscope's equatorial moment of

inertia;  $H$  is the angular momentum of the gyroscope;  $p$  and  $q$  are the projections of the gyroscope's angular velocity on the  $x$  and  $y$  axes;  $r'$  is the projection of the angular velocity for trihedron  $x, y, z$  upon the  $x$  axis;  $M_x, M_y$ , and  $M_z$  are the moments of forces about the axes of the moving trihedron.

In the precessional (or elementary) theory, it is assumed that the magnitude of the gyroscope's kinetic moment is approximately equal to  $H$  and is directed along the axis of the gyroscope. Then, the following equation is obtained:

$$Hq = M_x \quad -Hp = M_y \quad \dot{H} = M_z \quad [2]$$

Let axes  $\xi, \eta$ , and  $\zeta$  be approximated graphically. The position of the gyroscope's axes in this system of coordinates will be defined by angles  $\alpha$  and  $\beta$  as indicated in Fig. 1.

Translated from Uchenye Zapiski (Leningradskogo Universiteta, Seriya Matematicheskikh Nauk (Scientific Notes of Leningrad University, Mathematical Science Series), **35**, no. 280, 25-30 (1960). Translated by Primary Sources, New York.